Quantum Recurrent Neural Networks for Time Series Prediction

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Some Machine Learning algorithms, like Recurrent Neural Networks (RNNs), analyse time series to predict unknown values of variables in a complex system. When dealing with multi-layer networks and broad series, some issues, such as overfitting or memory losses, arise. Several approaches intend to address them, for example, the Long Short-Term Memory (LSTM) cell. Despite these approaches, learning from multivariate-complex systems is still a challenge and requires networks with many non-linear terms, expensive to compute on classical devices.
Quantum Computation emerges as a promising approach to tackle complex problems more efficiently since it allows to compute non-linear terms in a high dimensional space without spending exponential resources. We propose a Quantum RNN (QRNN) model as a first step towards multivariate time series forecasting. The core of QRNN is a parametrized quantum circuit that iteratively exchanges information, but, at the same time, it keeps memory from past data.

1. Motivation

Some applications of Time Series

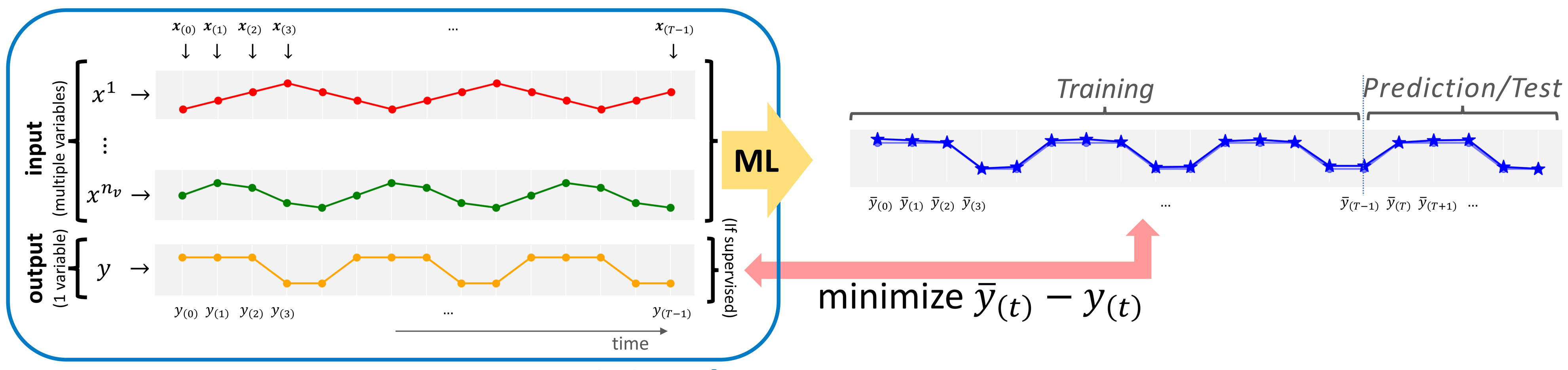
- **Finance**
  - Stock prices
  - Financial crashes
- **Meteorology-geology**
  - Weather forecasting
  - Earthquake prediction
- **Medicine**
  - Electroencephalography
- **Artificial Intelligence**
  - Natural language
  - Speech recognition
- **Industry**
  - Data from sensors

Common issues

- Large sequences
- High non-linearity
- High dimensionality

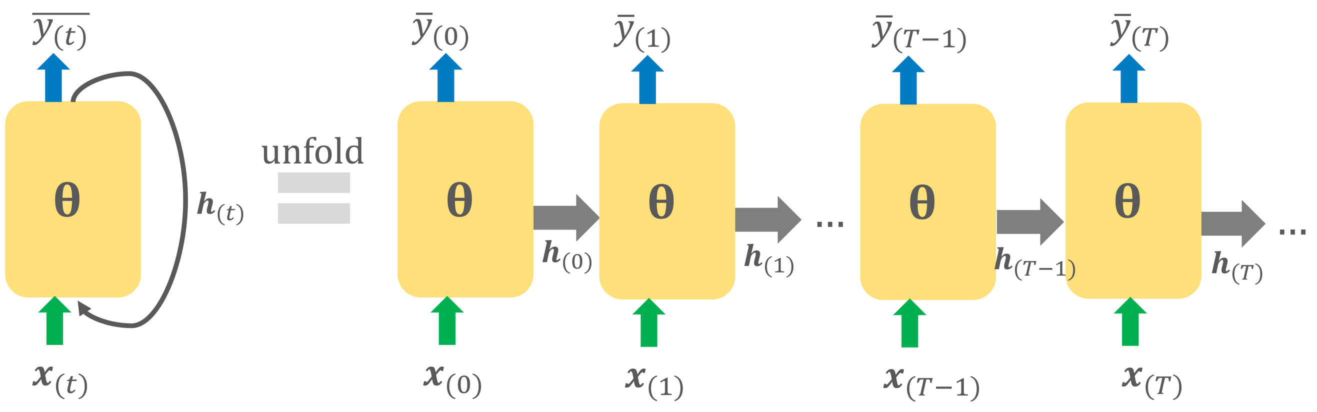
2. Multivariate time series

Our final **objective** is to have a Machine Learning (ML) algorithm that models **multivariate time series**.



Training data

3. The Classical RNN model



The supervised RNN cell receives a multivariate input at each time and returns:

- an estimation as **output**, $\overline{y_{(t)}}$,
- a **hidden state**, h.

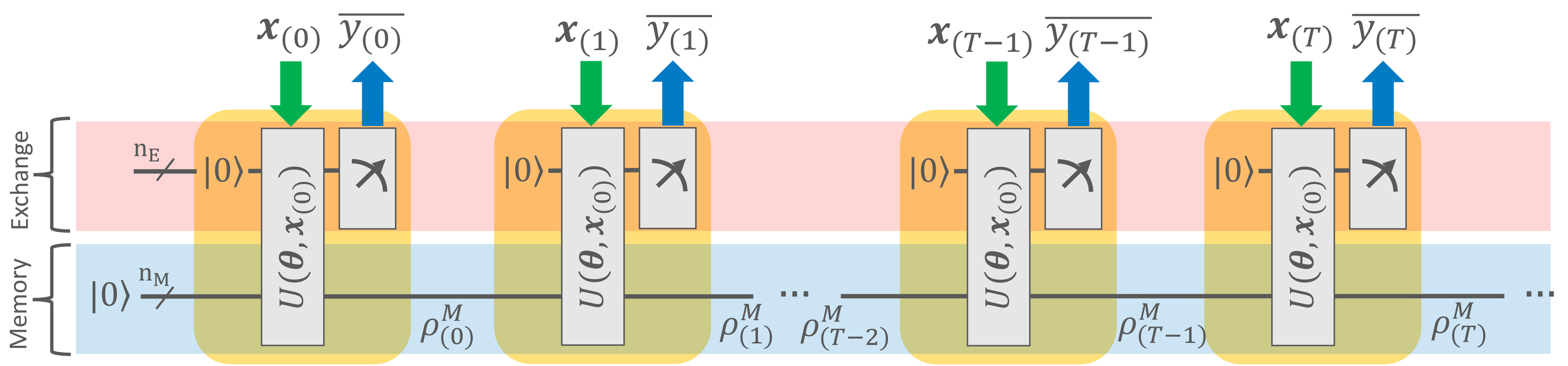
The latter is reintroduced inside the cell recurrently.
We show the cell and its unfolded representation through time [1].

4. The QRNN circuit structure

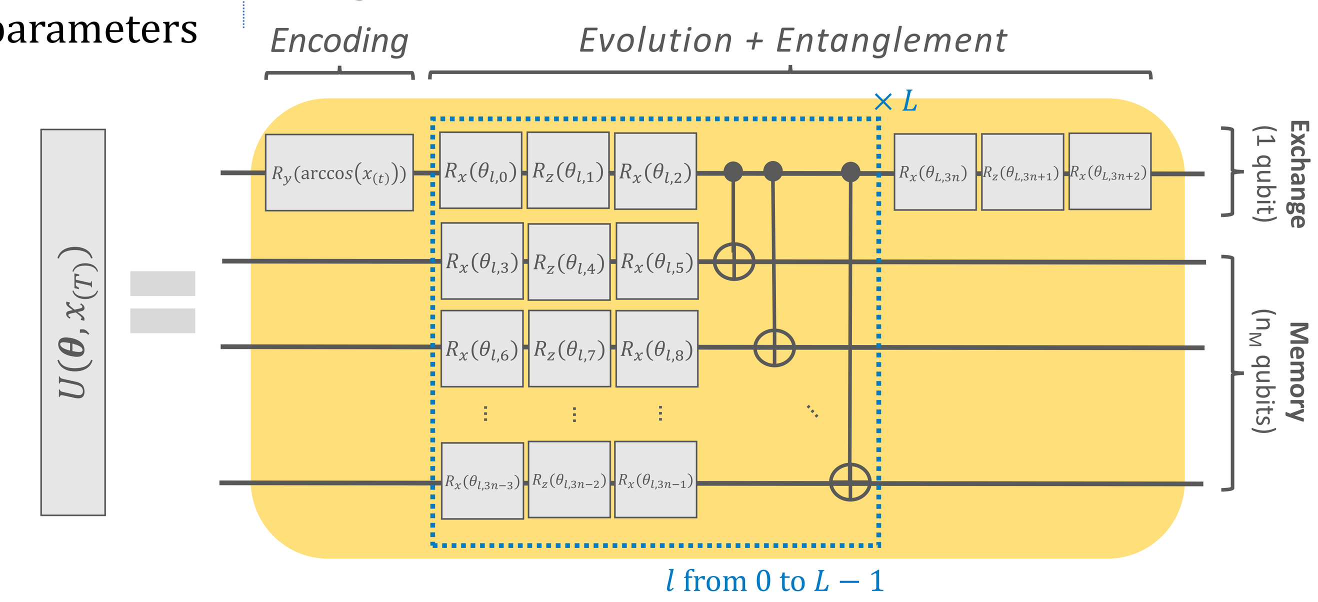
The QRNN circuit is inspired in the classical RNN, by dividing the qubits into two groups [2,3]:

- **Exchange register (E)**: in each frame, it receives classical data and is finally measured.
- **Memory register (M)**: is never measured, keeping information from all previous inputs.

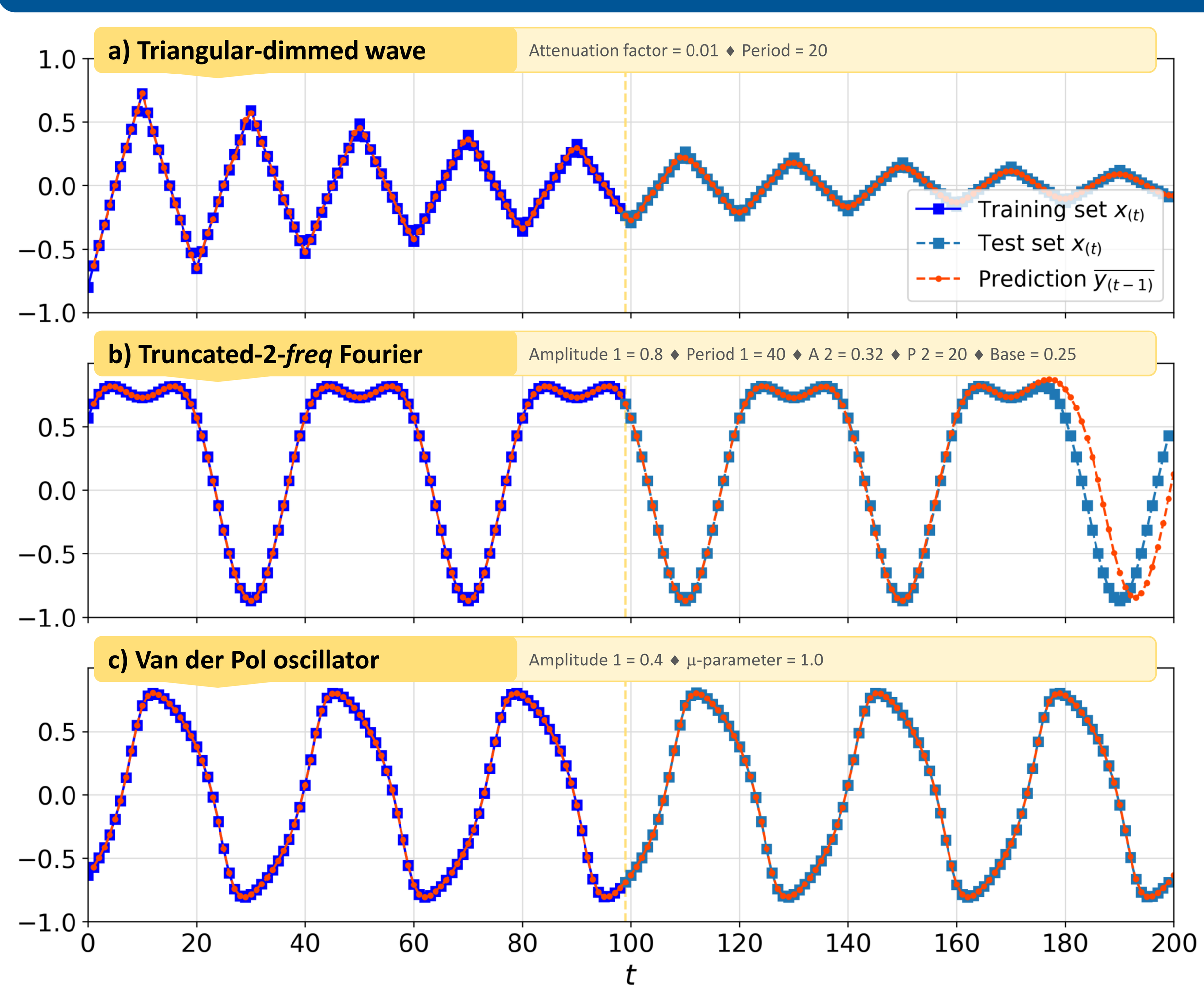
The unitary U encodes data into the circuit, applies an evolution depending on the set of parameters $\boldsymbol{\theta}$ and entangles qubits from E to M, transporting information between the two registers.



Our unitary U for univariate inputs is similar to the proposal in ref. [4]. It is a hardware-efficient ansatz for gate-based quantum devices.



5. Results



We use our QRNN model to **predict** future values from a known series. At a time t, the network takes the value $x_{(t)}$ and predicts the next one in the sequence, i.e., $\overline{y_{(t)}} = x_{(t+1)}$.

- **Cost function**: root mean square error (RMSE) between the network's outputs $\overline{y_{(t)}}$ and the references $y_{(t)}$.
- **Parameters**: the set $\boldsymbol{\theta}$ for rotation gates + a scaling factor c. $\overline{y_{(t)}} = c \cdot \langle Z \rangle_{(t)}$
- **Optimization**: minimization of the RMSE by **BFGS** method; random initialization of $\boldsymbol{\theta}$.

Here, three different cases as a test for the model. The circuit is ideally simulated.

- $n_M = 3$ qubits
- $L = 4$ layers
- $N_\theta = 52$ parameters

6. Perspectives

Conclusions for this work:

- The presented QRNN model can predict sequences of one-variable series.
- With a small number of qubits we achieve a good convergence.

Next steps:

- Data re-uploading to increase the expressivity of the encoding [5,6].
- Algorithm for multivariate series.
- Simulation with noise and sampling.
- Explore other optimization techniques: stochastic and genetic algorithms.

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